

Lois discrètes usuelles : formulaire

	Notation et paramètres	Loi de X	Fonction génératrice	$\mathbb{E}(X)$	$\mathbb{V}(X)$
Loi uniforme	$\mathcal{U}(\llbracket 1, n \rrbracket)$ $n \in \mathbb{N}^*$	$X(\Omega) = \llbracket 1, n \rrbracket$ $\forall k \in \llbracket 1, n \rrbracket, \mathbb{P}(\{X = k\}) = \frac{1}{n}$	$\forall t \in]-\infty, +\infty[,$ $G_X(t) = \begin{cases} \frac{1}{n} \frac{t - t^{n+1}}{1 - t} & \text{si } t \neq 1 \\ 1 & \text{si } t = 1 \end{cases}$	$\frac{n+1}{2}$	$\frac{(n-1)(n+1)}{12}$
	$\mathcal{U}(\llbracket a, b \rrbracket)$ $(a, b) \in \mathbb{N}^2$ $b \geq a$	$X(\Omega) = \llbracket a, b \rrbracket$ $\forall k \in \llbracket a, b \rrbracket, \mathbb{P}(\{X = k\}) = \frac{1}{b-a+1}$	$\forall t \in]-\infty, +\infty[,$ $G_X(t) = \begin{cases} \frac{1}{b-a+1} \frac{t^a - t^{b+1}}{1 - t} & \text{si } t \neq 1 \\ 1 & \text{si } t = 1 \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)(b-a+2)}{12}$
Loi de Bernoulli	$\mathcal{B}(1, p)$ $p \in]0, 1[$	$X(\Omega) = \{0, 1\}$ $\mathbb{P}(\{X = 1\}) = p$ et $\mathbb{P}(\{X = 0\}) = 1 - p$	$\forall t \in]-\infty, +\infty[, G_X(t) = (1-p) + pt$	p	pq
Loi binomiale	$\mathcal{B}(n, p)$ $n \in \mathbb{N}^*$, $p \in]0, 1[$	$X(\Omega) = \llbracket 0, n \rrbracket$ $\forall k \in \llbracket 0, n \rrbracket, \mathbb{P}(\{X = k\}) = \binom{n}{k} p^k q^{n-k}$	$\forall t \in]-\infty, +\infty[, G_X(t) = ((1-p) + pt)^n$	np	npq
Loi géométrique	$\mathcal{G}(p)$ $p \in]0, 1[$	$X(\Omega) = \mathbb{N}^*$ $\forall k \in \mathbb{N}^*, \mathbb{P}(\{X = k\}) = p q^{k-1}$	$\forall t \in]-\frac{1}{1-p}, \frac{1}{1-p}[, G_X(t) = \frac{pt}{1 - (1-p)t}$	$\frac{1}{p}$	$\frac{q}{p^2}$
Loi de Poisson	$\mathcal{P}(\lambda)$ $\lambda > 0$	$X(\Omega) = \mathbb{N}$ $\forall k \in \mathbb{N}, \mathbb{P}(\{X = k\}) = e^{-\lambda} \frac{\lambda^k}{k!}$	$\forall t \in]-\infty, +\infty[, G_X(t) = e^{\lambda(t-1)}$	λ	λ

$$G_X : t \mapsto \mathbb{E}(t^X) = \sum_{k=0}^{+\infty} \mathbb{P}(\{X = k\}) t^k$$

$$\mathbb{E}(X) = G'_X(1)$$

$$\mathbb{E}(X(X-1)) = G''_X(1)$$

$$\mathbb{V}(X) = G''_X(1) + G'_X(1) - (G'_X(1))^2$$